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Skyrmions and composite fermions in the limit of vanishing Zeeman energy

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Abstract. We describe the properties of a strongly interacting two-dimensional (2D) electron gas in high magnetic fields whose properties can be described in terms of the formation of composite fermions at fractional Landau level occupancy. For a Landau level occupancy $\nu = 1$ the excitations correspond to spin waves for high g-factors and magnetic fields, but when the g-factor is reduced close to zero by the use of hydrostatic pressure there is strong evidence for the formation of skyrmions. Studies of composite fermions in the limit of vanishing Zeeman energy also suggest that skyrmionic excitations can occur for composite fermions.

The quantum Hall effect and fractional quantum Hall effect (FQHE) which are observed for a two-dimensional system in high magnetic fields are two of the most fascinating examples of the importance of electron–electron interactions in condensed matter physics [1, 2]. In the last few years it has been realized that a good way to understand the FQHE is through the introduction of composite fermion (CF) particles [3], where the strong electron–electron interaction causes new excitations of the system in which an electron becomes bound to a pair of flux quanta. The basic idea is to use a Chern–Simons gauge transformation to describe the electron–electron interactions with mean field, $B_M = 2n_e\Phi_0$, which corresponds to two flux quanta per electron. Moving away from $\nu = 1/2$ the composite fermions experience a residual, or effective, magnetic field (B^*) given by the difference of the applied external field, and the mean field used in the formation of the composite fermions. The effective field is thus

$$B^* = B - 2m\Phi_0 n_e.$$

This B^* leads to a quantization of the CF energies and motion in Landau levels with the same degeneracies as electrons, and energy separations given by some composite fermion cyclotron energy

$$E^* = \hbar\omega_c^* = \frac{\hbar e B^*}{M^*}$$

where we now have to define an appropriate effective mass M^* which will describe the energy gaps. The composite fermions show a Shubnikov–de Haas effect (resistivity minima) and a corresponding quantum Hall effect, remembering that CF features can arise for both

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positive and negative values of effective field (i.e. as we move both upwards and downwards away from $\nu = 1/2$). The oscillations occur at magnetic fields given by

$$B^* = \frac{\Phi_0 n_e}{\nu^*}$$

where $\nu^* = p$, an integer. This corresponds to total external fields of

$$B = 2m\Phi_0 n_e \pm B^* = \left(\frac{2m \pm 1}{p}\right) n_e \Phi_0$$

and occupancies of

$$\nu = \frac{p}{(2mp \pm 1)}.$$

In the simplest case ($m = 1$, $p = 1, 2, 3, \ldots$) this gives the series of fractions 1/3, 2/5, 3/7 for positive effective field values, and 1, 2/3, 3/5, 4/7, ..., for the negative effective fields. These are precisely the occupancy values for the strongest features observed in the FQHE.

The energy gaps and hence the effective masses of the particles have been measured in GaAs/GaAlAs heterojunctions by analysing the temperature dependence of the resistivity oscillations with the Lifshitz–Kosevich (Ando) formula [4]. The mass has been found to be a slowly increasing function of n_e, with typical values of approximately ten times the conduction band-edge electron mass [5, 6]. There is no unique n_e dependence for all fractions, but the mass values are found to fall in pairs, corresponding to states with a common numerator p, for example 2/3 and 2/5. These have equal numbers of occupied CF Landau levels, but occur on either side of $\nu = 1/2$ with effective fields in opposite senses. This provides a simple demonstration of the symmetry of the states about $\nu = 1/2$ which is consistent with the CF model, rather than that of particle-hole conjugation where states of the common denominator q (e.g. 1/3 and 2/3) look similar. The measurement of M^* is equally a measurement of the CF cyclotron energy $E^* = \hbar e B^*/M^*$. This is equivalent to what would previously have been known as the FQHE energy gap Δ, for a sample with infinitely narrow levels. This gap E^*_ν is found to be a single function of B^* [5, 6] for all the samples studied as shown in figure 1. The increase of M^* with field necessarily means that E^* shows a sub-linear increase with effective field. For values of B^* above ~2 T it has been found that $E^* = a\sqrt{B^*}$, where $a = 3.3$ K T$^{-1/2}$.

The FQHE is the result of a many-body Coulomb interaction and so theoretically [3] we would expect the energy gaps to scale with a relation of the form $E^*_\nu = C_\nu E_c$, $E_c = e^2/(4\pi\varepsilon\varepsilon_0 l_0)$, where l_0 is the cyclotron radius (proportional to the interparticle spacing) and C_ν is a fixed coefficient, different for each fraction. Halperin *et al* [3] have used this relationship and argued on dimensional grounds that the high field limit of M^* should show a $(B)^{1/2}$ dependence on carrier density through l_0. This corresponds to a $(B)^{1/2}$ dependence for both E^* and M^* for any given fraction; however, the results suggest that there is not a single functional dependence on n_e but instead on B^*.

When $\nu = 1$, we move to a rather different example of the influence of Coulomb interactions. Taking account of electron spin this occupancy corresponds to a single completely filled spin level, which will act as an itinerant (quantum Hall) ferromagnet. At this point the energy gap is dominated by the Coulomb exchange energy [7], which is considerably larger than the single particle Zeeman energy [8]. The excitations of the system may be associated with either single spin flips which generate spin waves [9], or if the Zeeman energy is not too large, they may be spin-textured chiral solitons which become skyrmions in the limit of $g = 0$ [10, 11]. Such an excitation is shown schematically

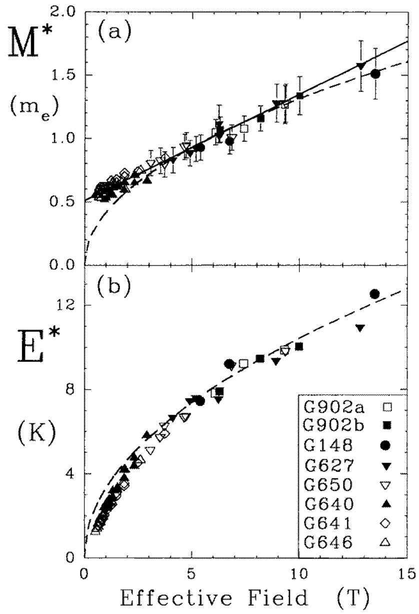

Figure 1. The dependence of the CF effective mass and energy gap E^* on the effective field B^*.

in figure 2. A central reversed spin is surrounded by rings of spin that gradually cant over until at the edge they are aligned with the external magnetic field. The essential differences between this two-dimensional excitation and a spin-wave are that it is charged, the net spin may be considerably greater than one, and on a path taken around the central spin there will be a net change of spin orientation equivalent to a winding number of unity. For systems with a finite Zeeman energy the skyrmions have finite size that can be characterized by the number of reversed spins, R, contained in the skyrmion. They have been detected optically from the degree of spin polarization in nuclear magnetic resonance [12] and photoluminescence experiments [13]. Both measurements suggest that $R \sim 7$. The parameter which determines whether skyrmions or spin waves will be the lowest lying excitations is $\eta = g^*\mu_B B/E_c$. The crossover is calculated to be at $\eta = 0.054$ [11]; however, it is only below $\eta < 0.01$ that a very significant energy difference from single spin flips occurs. Since $\eta \approx (B)^{1/2}$ skyrmionic excitations are expected to be favoured at low magnetic fields and small g-factors. To date two transport measurements have inferred the existence of skyrmions. Increasing η by tilting the magnetic field suggested a seven spin excitation for η 0.01 [14]. In a narrow quantum well where g^* is reduced due to conduction band non-parabolicity effects, it was decreased further by applying hydrostatic pressure so that it became zero at ~4.8 kbar and thus revealed a reduction in energy gap when $\eta < 0.002$ consistent with 33 spin flips [15].

Hydrostatic pressure is a very convenient parameter which allows us to vary the g-factor continuously [16] and thus look for evidence of skyrmion formation and changes in skyrmion size. The reduction in g-factor is due to the change in contribution to the spin orbit splitting caused by the change in band gap. The g-factor has been calculated using $\boldsymbol{k}\cdot\boldsymbol{p}$ theory [17] and may be approximated by the following expression

$$g = 2 - 19\,300\left(\frac{1}{1519 + \hbar\omega_c + 10.7P} - \frac{1}{1860 + \hbar\omega_c + 10.7P}\right) - 0.12$$

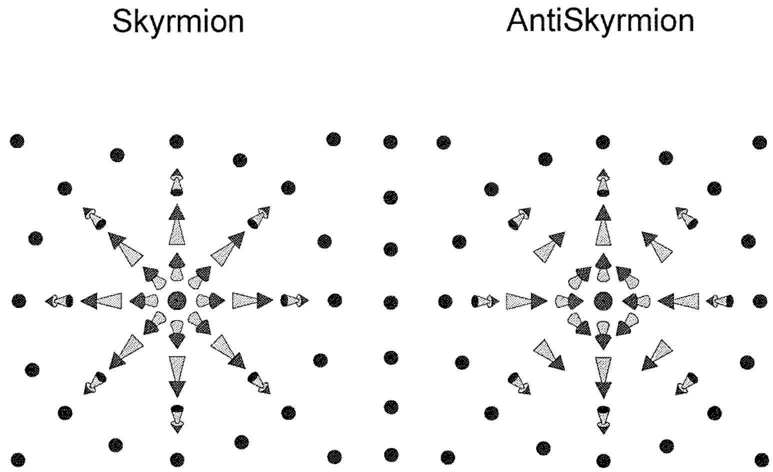

Figure 2. A schematic picture of the spin alignments in a skyrmion and an anti-skyrmion for a size comparable to those found in experiments.

where P is the pressure in kbar, and all of the far-band terms have been assumed to stay constant. This predicts a g-factor which will pass through zero in the region of 18 kbar, with the exact value somewhat dependent on the carrier density and hence the magnetic field at which the occupancy feature is being studied.

We examine the possibility of skyrmion formation in different regimes of Zeeman energy by looking at values of the energy gaps E_ν^* which have been extracted by fitting the temperature dependence of the resistivity minima to the Liftshitz–Kosevich (LK) formula, which accounts for the effects of the thermal smearing of the Fermi function. In this formula $\Delta\rho_{xx}/\rho \propto X/\sinh X$, where $X = 2\pi^2 k_B T/E^*$. This procedure, described in more detail in [6], has the advantage over finding activation energies from an Arrhenius plot that, first, it measures the gap between level centres not the mobility gap, and so is less sensitive to changes in disorder and second, an accurate zero of resistance is not required, which avoids any problems of parallel conduction and means that especially low temperatures are not required. A possible disadvantage of the LK method is that the majority of measurements are made in a temperature range in which the system may not remain totally spin polarized. The accuracy of the LK fitting procedure has been tested by considering the energy gaps at even integers which were found to be within 1% of the expected single particle cyclotron energy. In general, the odd occupancy data do not always fit the LK formula quite as well but we would expect the results to be accurate to 10%.

The activation energy Δ was also measured from an Arrhenius plot of $\rho_{xx} = \rho_0 \exp(-\Delta/2kT)$. By contrast this only uses data at the lowest temperatures. A comparison of the results from the two techniques can be seen in figure 3, which shows the energy gaps E^* deduced from the LK method and Δ from an Arrhenius plot in a sample with a 200 Å spacer layer, for pressures in the critical range 10–20 kbar where the g-factor approaches zero. The difference between the two values is due to Landau level (LL) broadening Γ. Provided the density remains unchanged this should be constant such that $E^* = \Delta + \Gamma$. As the gap becomes small the LLs overlap, no well developed resistivity zero is observed and the activation behaviour collapses. Consequently, the values deduced from the Arrhenius plots become highly questionable. Above 17 kbar even the LK fits fail systematically. This may be due in part to the fact that for the two highest pressures the maximum density which

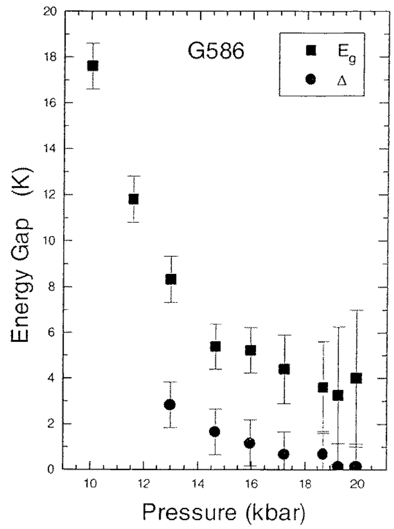

Figure 3. Energy gap at $\nu = 1$ for a sample with a density of $\sim 0.5 \times 10^{15}$ m^{-2}, measured from the LK formula (E_g) and from activated resistivity (Δ).

can be achieved by prolonged illumination is significantly lower than for lower pressures. At low temperatures the minima do not become zero. Additionally at the high temperature end of our data range the resistivity shows an unusually slow temperature dependence, which would be interpreted as a very large energy gap if the LK formula were still valid. The values of the energy gaps shown in figure 3 are for temperatures below this deviation from the LK formula, but those at the highest pressures must still be regarded as relatively uncertain. Notwithstanding these qualifications the gap at $\nu = 1$ clearly decreases as pressure is increased. There is also some evidence from the higher temperature traces that it reaches a minimum at 18 kbar and beyond this pressure the gap recovers again, although the low-temperature resistivity zero is not recovered. The existence of a symmetry about 18 kbar, although limited, is good evidence that the g-factor has really passed through zero at the pressure predicted by kp theory and indeed changed sign at higher pressures.

For pressures up to around 14 kB, and values of $\eta > 0.005$ we see [18] no evidence of skyrmion formation. Analysing the energy gaps at both $\nu = 1$ and 3 using the Lifshitz–Kosevich formula we find, as shown in figure 4 for two samples for hydrostatic pressure up to 14 kB, that the energy gap scales directly with Coulomb energy, with a value of $C_1 = 0.21$. The energy gaps are typically more than twenty times larger [8] than the single particle Zeeman energy and this behaviour is consistent with the excitation of simple spin waves up to the ionization limit, although it should be noted that the value of C_1 is considerably smaller than the theoretical estimate of $(\pi/2)^{1/2}$ for an ideal 2D system [9].

When the pressure is increased the carrier density falls and the Zeeman energy decreases sufficiently to show evidence of skyrmion formation [18]. Figure 5 shows a typical series of low-temperature (40 mK) measurements of resistivity for densities of order 0.5×10^{15} m^{-2} which have been measured as a function of increasing pressure, with the traces normalized to remove the effects of small changes in electron density. The minima at $\nu = 1$ weaken progressively as the energy gap decreases. In figure 6 we plot the energy gaps measured from LK analysis and scaled by the Coulomb energy, E_c, as a function of the normalized Zeeman energy η. For larger values of η the normalized gap is constant, corresponding

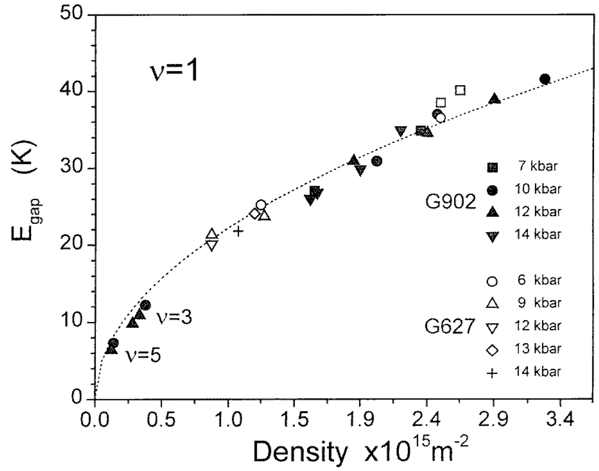

Figure 4. Energy gaps as a function of density at $\nu = 1$.

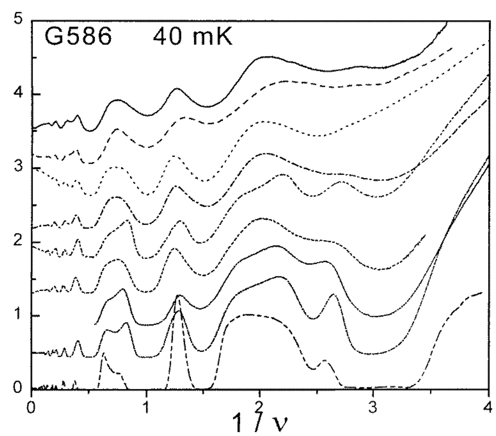

Figure 5. Resistivity as a function of occupancy in a series of traces from 10 (lowest trace) to 20 kbar (upper trace) in a GaAs/GaAlAs heterojunction at 40 mK.

to a value of $C_1 = 0.21$ as found earlier, but there is a sudden fall beginning around $\eta = 0.005$ with a very clear minimum. The slope around this minimum ($R = \mathrm{d}[E^*/E_c]/\mathrm{d}\eta$) corresponds to $R \sim 36$. This is clear evidence for the existence of skyrmions for values of $\eta < 0.005$, in good agreement with the results reported recently by Maude *et al* [15]. Theory suggests that skyrmion size should increase continuously as $|\eta|$ is reduced, so this value of $R = 36$ can only be taken as an average or limiting value. Kamilla *et al* [19] estimated the number of reversed spins in an anti-skyrmion by minimizing its energy $E(R)/E_c = 0.313 + 0.23\exp(-0.25R^{0.85}) + \eta R$. This shows an anti-skyrmion with 18 reversed spins (i.e. 36 in the pair excitation) would occur at $\eta = 0.0017$ which falls right in the middle of our data range, and that R falls to 11 by $\eta = 0.005$. However, the minimum energy of $0.313E_c$ at $\eta = 0$ which corresponds to a pair gap of $0.627E_c$, is very much larger than the $0.04E_c$ observed experimentally. Another qualitative difference from the theory is that instead of the cusp which would result if $R \to \infty$ as $g \to 0$ the minimum of figure 4

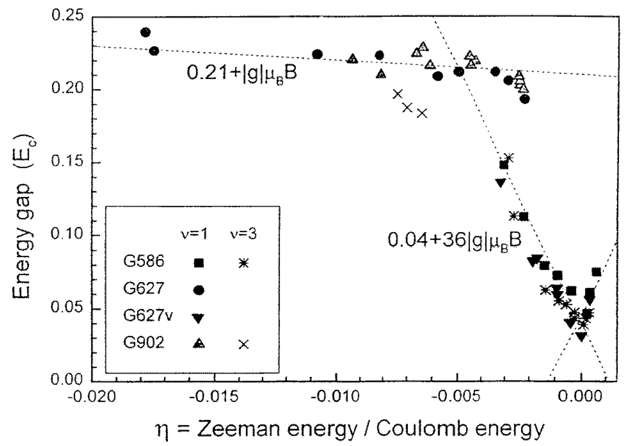

Figure 6. The energy gaps at $\nu = 1$ (full points), 3 (open points) for several different GaAs/GaAlAs samples as a function of Zeeman energy. Both axes are scaled by the Coulomb energy E_c. The dotted line with unit gradient shows the energy to create a single exciton. The dashed lines have gradients of ± 36 corresponding to skyrmion excitation.

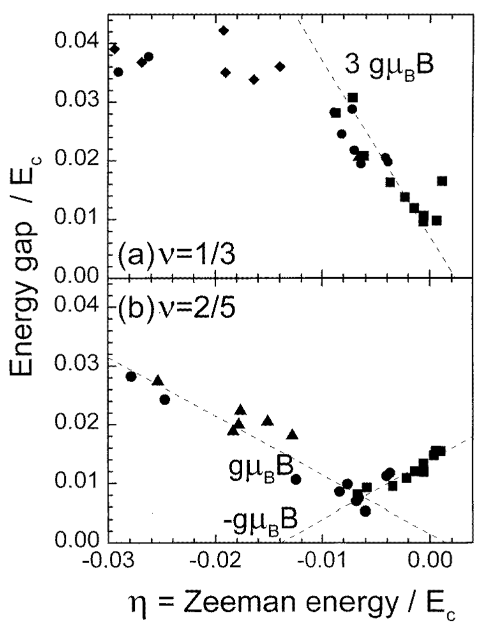

Figure 7. (a) Energy gap at $\nu = 1/3$ for several different samples studied as a function of the Zeeman energy. Both axes are scaled by the Coulomb energy E_c. The line shows the energy required to flip 3 spins. (b) The energy gap at $\nu = 2/5$. The slope of the lines now corresponds to a single spin flip.

is more rounded, as found in [15]. This may be explained by long-range disorder limiting skyrmion size. At the density of 0.44×10^{15} m^{-2} an 18 spin skyrmion would have a radius of 1140 Å which is already larger than the spacer layer thickness that usually determines the scale of the disorder potential in modulation doped structures.

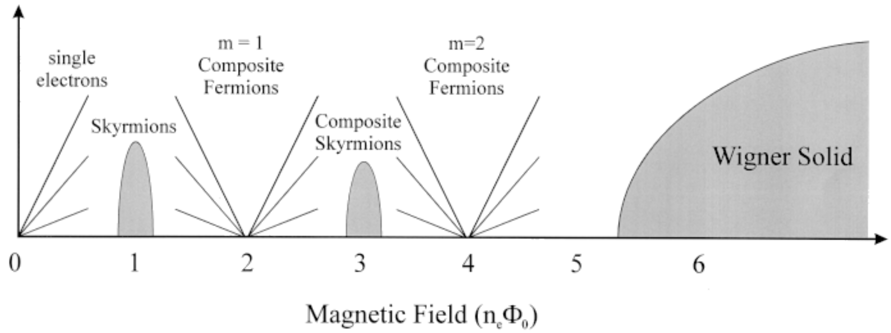

Figure 8. A schematic view of the different ground states and excitations which occur for a 2D electron system in high magnetic fields.

We move now to the effects of varying the g-factor on the composite fermion states [20]. The introduction of spin into the composite fermion picture has a number of consequences, leading to the possibility of both polarized and unpolarized ground states [21]. For the most fundamental of the FQHE states, at $\nu = 1/3$, the system is an itinerant ferromagnet of composite fermions and is therefore the CF analogue of $\nu = 1$, where we might expect the existence of Skyrmionic excitations also. Figure 5 also shows how the FQHE states change as a function of pressure [20]. There is an obvious disappearance of the $\nu = 1/3$ state as the g-factor vanishes, while the states at 2/3 and 2/5 remain strong. A study of the scaled energy gaps of the fractions, shown in figure 7, shows that the 1/3 state has a value of energy gap corresponding to $C_{1/3} = 0.04$ for $\eta > 0.01$ but that it decreases its gap rapidly as the Zeeman energy falls below this. This suggests that for small g-factors the excitations of the system are skyrmionic composite fermions, with an apparent size of $\sim$3 spins deduced from the slope of the gap, in good agreement with recent theoretical estimations [19].

Analysis of the gap for the 2/5 state suggests that this state has a minimum gap energy at $\eta \sim 0.006$. This corresponds to the point where the 2/5 state changes from a polarized to an unpolarized ground state, but still maintains a finite gap at the transition. This behaviour is similar to that observed previously for 2/3 at very low magnetic fields by tilting the sample [22].

In conclusion, therefore, we can say that we have a variety of examples in which there exists a family of different ground states and excitations for the 2D electron system in high magnetic fields. This is shown schematically in figure 8, which shows the different field ranges in which the various states are the preferred description of the system, including the high field limit of a magnetic field induced Wigner solid.

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